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# Mesoscopic Simulation of Non-Newtonian Natural Convection in a Stenotic Artery

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## Abstract

Lattice Boltzmann method is used to simulate natural convection in blood flow through stenotic artery. The problem of natural convection in stenotic artery is considered and the influence of elevated temperature and material properties is studied on the flow properties. A porous like square stenotic medium in a human artery with fluid (blood) at the left wall of the geometry and the north wall kept to a normalized temperature of 1.0 is considered. The flow properties like velocity profiles, streamlines, temperature profiles and the rate of heat transfer are then studied with respect to the material properties like porosity and permeability and flow parameters like Rayleigh number and power law index.

***Keywords:*** *Non-Newtonian, porous media, stenosis, lattice Boltzmann method, natural convection*

## I. INTRODUCTION

Numerical simulation has a potential ability to assist developments in medical research by providing a reliable alternative for decision making by not only providing a low cost decision making tool but by helping to plan for a medical procedure for future advances. As a result, a wide range of biomedical research focuses on numerical techniques that can aid in these decision support tools. For instance, simulation of blood flow in cardiovascular diseases like aneurysm, thrombosis, stenosis etc. can lead to a systematic understanding of growth of the disease so as to take appropriate measures to sustain it. It is known that blood flow in these circumstances either aggravates the disease or helps in a systematic cure by a controlled procedure. A detailed rheology of blood is discussed in [1-4]. Various models like power-law model, Casson model, K-L model, Cross model and Carreau-Yasuda model have been proposed to represent blood flow [2, 5], of which Casson model has been widely used in the literature. However, the disadvantage of this model is the limited validity of shear rate range which is overcome by Carreau-Yasuda model [6]. While most of research considers blood to be Newtonian, at low shear rates it resembles to exhibit non-Newtonian behavior and its viscosity happens to be a function of the shear rate. This paper considers blood to be non-Newtonian shear thinning fluid considering practical implications. A porous like square stenotic medium in a human artery with fluid (blood) at the left wall of the geometry and the north wall kept to a normalized temperature of 1.0. The flow properties like velocity profiles, streamlines, temperature profiles and the rate of heat transfer are then studied with respect to the material properties and flow parameters like Rayleigh number and power law index. Porosity is varied from 0.1 to 0.7 whereas is fixed at $10^{-3}$. The power law index is varied from 0.5 to 1 to consider shear-thinning behaviour of blood and Prandtl number Pr is taken as 4, while Rayleigh number Ra is varied from $10^3$ to $10^5$.

## II. NUMERICAL METHOD AND IMPLEMENTATION

### A. Governing Equations

The mathematical model for natural convection in porous media can be expressed by the continuity equation, the Brinkman–Forchheimer equation, and the energy equation [7-8]

$$\nabla . u = 0 \tag{1a}$$

$$\frac{\partial u}{\partial t} + (u.\nabla)\left(\frac{u}{\varepsilon}\right) = -\frac{1}{\rho}\nabla(\varepsilon p) + v_e \nabla^2 u + F \tag{1b}$$

$$\frac{\partial(\rho e)}{\partial t} + \nabla.(\rho u e) = \lambda_{eff}\nabla^2(\rho e)) \tag{1c}$$

The fluid is modeled by a single-particle distribution function $f_i(x,t)$ governed by a lattice Boltzmann equation [7-8]

$$f_i(x + e_i dt, t + dt) - f_i(x,t) = \frac{f_i^{eq}(x,t) - f_i(x,t)}{\tau} + dtF_i \tag{2}$$

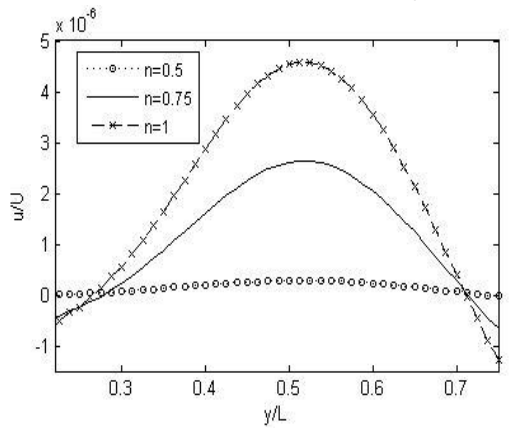
The force term is given by

$$F_i = w_i \rho \left(1 - \frac{1}{2\tau}\right)\left[\frac{e_i.F}{c_s^2} + \frac{uF:(e_i e_i - c_s^2 I}{\varepsilon c_s^2}\right] \quad (3)$$

### B. Energy Equation

The thermal lattice BGK model is given by [11]

$$g_i(x + e_i dt, t + dt) - g_i(x,t) = \frac{g_i^{eq}(x,t) - g_i(x,t)}{\tau_g} \quad (4)$$

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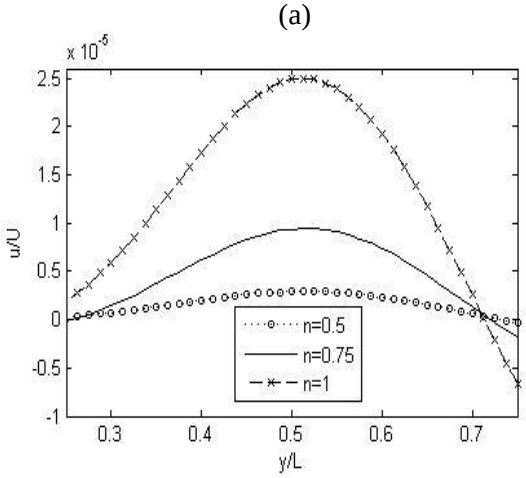


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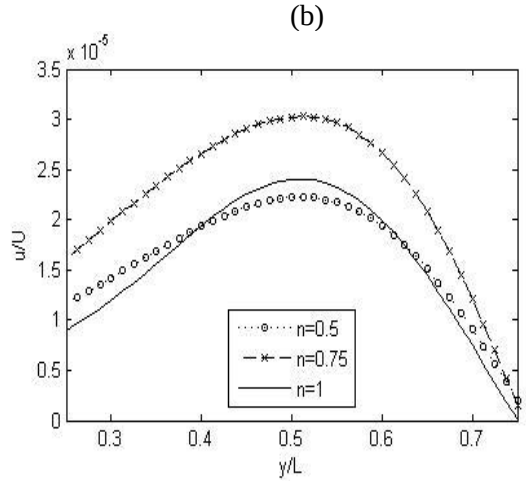



(c)

**Figure 1. u-velocity profiles for** $eps = 0.4$ **at various values of** $n$ **(a)** $Ra = 10^3$ **(b)** $Ra = 10^4$ **(c)** $Ra = 10^5$**.**

### C. Boundary Conditions and Implementation

Second order bounce back rule for non-equilibrium distribution function [14] is used to determine velocity on the four walls. For energy distribution function, second order extrapolation rule is used on the right wall and the energy distribution function at all other walls were determined the boundary conditions for all other walls were defined as [15].

Carreau-Yasuda model is used to represent non-Newtonian fluids given by [16]

$$\mu = \mu_0 + (\mu_0 - \mu_\infty) \Big/ \left(1 + a\left(\dot{\gamma}\right)^b\right)^{\frac{n-1}{b}} \quad (5)$$

## III. RESULTS AND DISCUSSION

For non-Newtonian fluids, Darcy's law for fluid flowing through a porous media is given by [17]

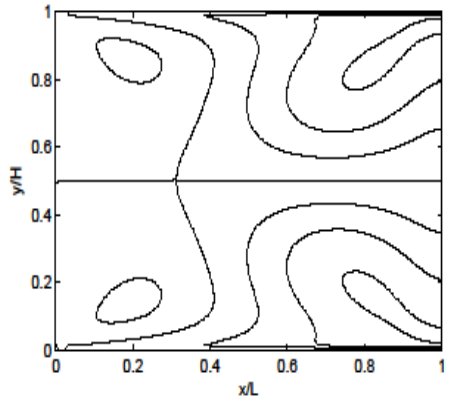
$$q = \left(\frac{K}{\mu_{eff}} \frac{\nabla p}{L}\right)^{1/n} \quad (6)$$

Thus, validity of the numerical procedure for non-Newtonian numerical simulation can be established by verifying that the plot of $q$ and $\nabla p = grad(p)$ is linear with slope $1/n$ as given in Table 1. The values of parameter for values 0.5, 0.75 and 1 of $\mu_0$ were identified as 0.1, 0.06 and 0.02, respectively, while $\mu_\infty$ was fixed at 0.001.

Parameters $a$ and $b$ were taken as 2 and 0.64, respectively. Boundary conditions based on non-equilibrium parts were implemented on velocity distribution function, while the unknown energy distribution functions were determined as

$$g_\alpha = T_w(w_\alpha + w_\beta) - g_\beta \quad (18)$$

The temperature difference between top wall and the fluid induces fluid flow inside the geometry which results in heat transfer. Various flow properties are recorded to quantify the influence of natural convection on these parameters. The u-velocity profiles in Fig 1 presents the variation in u-velocities with respect to $n$ for fixed values of $\varepsilon$ and Ra at the center half of the cavity. The profiles show a sinusoidal behavior with the highest velocity magnitude at geometric center of the cavity. The u-velocities were observed to increase in magnitude with $n$, although the basic trend remains the same. Influence of $\varepsilon$ and Ra is also observed on velocity profiles as $\varepsilon$ is increased from 0.4 to 0.7 and Ra form $10^3$ to $10^5$, with a similar behavior as in Fig 1. Plots in Fig 2-3 show the influence of these flow parameters on streamlines.

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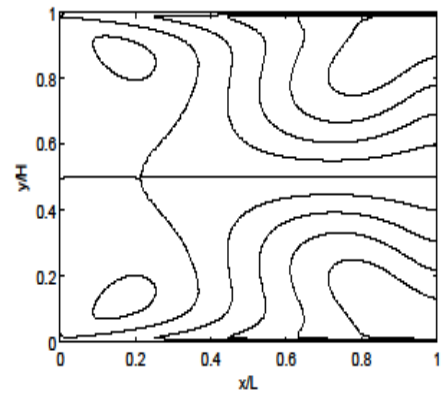


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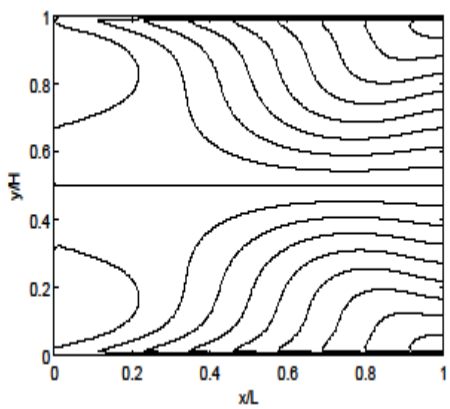
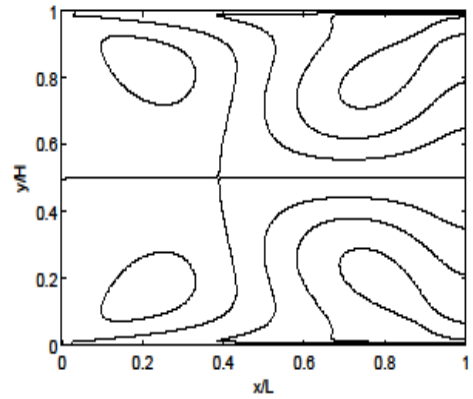


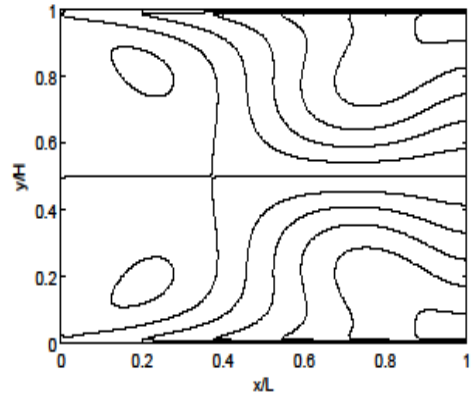

(c)

**Figure 2. Streamline plots for** $a = 0.5$ **and** $eps = 0.4$ **(a)** $Ra = 10^3$ **(b)** $Ra = 10^4$ **(c)** $Ra = 10^5$

Streamlines are symmetric as required for natural convection in a differentially heated geometry. At higher Ra values, flow strength is stronger as compared to lower values. Also, as $n$ increases from 0.5 to 1, circulation is observed to shift towards the left wall of the cavity. Formation of vortices can be observed for lower values of Ra implicating the flow circulation due to dominance of conduction over conduction. As the influence of convection start dominating, flow field expands to major part of the geometry.




(a)

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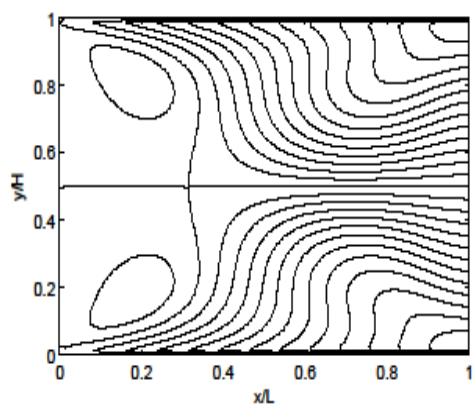


(c)

**Figure 3. Streamline plots for** $a = 0.75$**,** $eps = 0.4$ **(a)** $Ra = 10^3$ **(b)** $Ra = 10^4$ **(c)** $Ra = 10^5$

For lower values of Ra, vortices on the left of the geometry tend to occupy bigger space. However, these vortices shift towards left and disappear as flow is influenced by buoyancy effect for higher values of Ra. As $n$ increases from 0.5 to 1, flow field due to convection increases which is relevant form expansion of flow field from right to left. Variation in Nu values at hot wall are recorded in Table 1 and the average value of the relaxation parameter for various values of $n$ are recorded in Table 2.

The rate of heat transfer increases with an increase in porosity and decreases with an increase power law index.

**Table 1. Nu at hot wall for various values of $n$, $\varepsilon$ and Ra**

| $n$ | $\varepsilon = 0.4$ | | | $\varepsilon = 0.7$ | | |
|---|---|---|---|---|---|---|
| | Ra | | | Ra | | |
| | $10^3$ | $10^4$ | $10^5$ | $10^3$ | $10^4$ | $10^5$ |
| 0.5 | 1.3154 | 1.3169 | 1.3357 | 1.3179 | 1.3205 | 1.3391 |
| 0.75 | 1.3104 | 1.3157 | 1.3274 | 1.3132 | 1.3168 | 1.3299 |
| 1 | 1.3029 | 1.3140 | 1.3261 | 1.3039 | 1.3144 | 1.3264 |

**Table 2. Value of $\tau$ for various values of $n$, $\varepsilon$ and Ra**

| $n$ | $\varepsilon = 0.4$ | | | $\varepsilon = 0.7$ | | |
|---|---|---|---|---|---|---|
| | Ra | | | Ra | | |
| | $10^3$ | $10^4$ | $10^5$ | $10^3$ | $10^4$ | $10^5$ |
| **0.5** | 0.5956 | 0.5755 | 0.5287 | 0.5948 | 0.5735 | 0.5278 |
| **0.75** | 0.6021 | 0.5858 | 0.5421 | 0.6009 | 0.5847 | 0.5403 |
| **1** | 0.5600 | 0.5600 | 0.5600 | 0.5600 | 0.5600 | 0.5600 |

## IV. CONCLUSIONS

There is limited evidence for the efficacy of practices of naturopathy, and limited evidence to prove its ineffectiveness. With advanced medicinal technology and elevating cost, there is a need of significant research in this area, making this process available in India and attract good researchers to investigate the significance of naturopathy, and thus, hydrotherapy. These results shall give an insight into the medical process, mathematically, which will help in making significant improvements in the methods followed. This will help in developing these medical practices as an alternative to the existing medical processes to reduce the medical costs in India.

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